

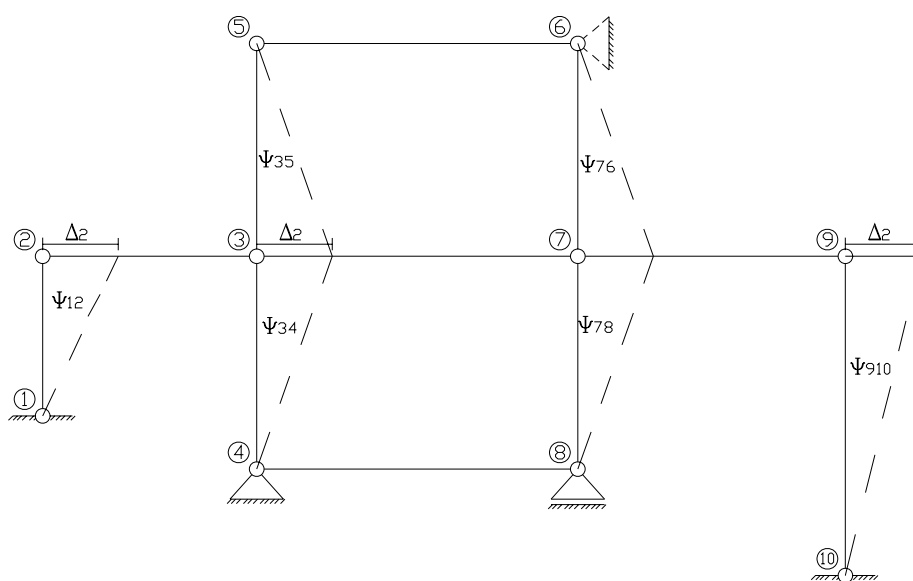
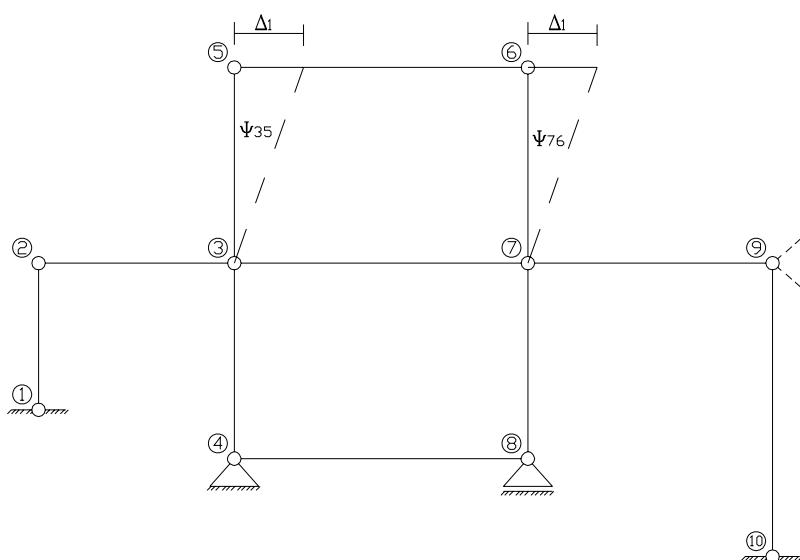
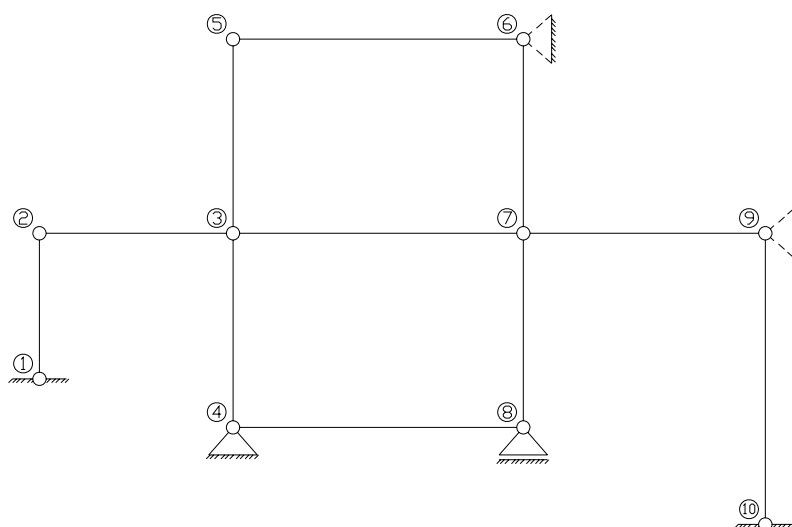
UNIVERZITET „DŽEMAL BIJEDIĆ“ U MOSTARU
GRAĐEVINSKI FAKULTET
PREDMET: TEORIJA KONSTRUKCIJA 2

PROGRAMSKI ZADATAK III

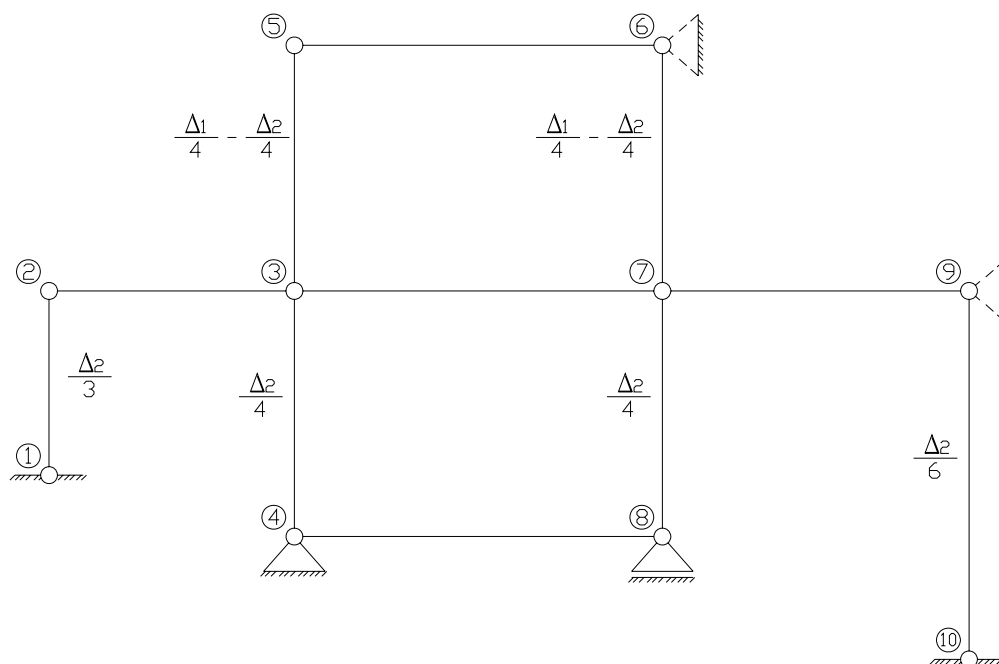
STUDENT: _
BR. INDEKSA: ____
ŠKOLSKA GODINA 2006/07

ZADATAK ZADAO:
v. ass.mr.RAŠID HADŽOVIĆ dipl.inž.grad.

Šema pomjeranja



Ukupno pomjeranja



Nepoznate:

$$\Delta_1, \Delta_2, \varphi_2, \varphi_3, \varphi_6, \varphi_7, \varphi_9$$

Momenti inercije

$$(S_1) \quad I_1 = \frac{0.3 * 0.3^3}{12} = 0.000675 m^4$$

$$(G_1) \quad I_2 = \frac{0.3 * 0.4^3}{12} = 0.0016 m^4$$

$$(G_2) \quad I_3 = \frac{0.4 * 0.6^3}{12} = 0.0072 m^4$$

$$(S_2) = (G_3) \quad I_4 = \frac{0.3 * 0.5^3}{12} = 0.003125 m^4$$

$$I_1 = 0.000675 m^4$$

$$I_2 = 2.37 I_1$$

$$I_3 = 10.67 I_1$$

$$I_4 = 4.63 I_1$$

konstante štapova

$$K = \frac{2EI}{l}$$

$$K_{12} = \frac{2EI_1}{l_1} = \frac{2EI_1}{3} = 0.667EI_1$$

$$K_{23} = \frac{2 * 2.37EI_1}{4} = 1.185EI_1$$

$$K_{23} = \frac{2 * 2.37EI_1}{4} = 1.185EI_1$$

$$K_{37} = \frac{2 * 10.67EI_1}{6} = 3.557EI_1$$

$$K_{56} = \frac{3 * 10.67EI_1}{6} = 5.335EI_1$$

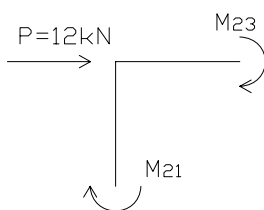
$$K_{43} = K_{35} = K_{78} = \frac{3EI_1}{4} = 0.75EI_1$$

$$K_{76} = \frac{2EI_1}{4} = 0.5EI_1$$

$$K_{79} = \frac{2 * 4.63EI_1}{5} = 1.852EI_1$$

$$K_{910} = \frac{2 * 4.63EI_1}{6} = 1.543EI_1$$

ČVOR 2.



$$M_{12} = K_{12}(2\varphi_1 + \varphi_2 - 3\Psi_{12}) + \eta_{12}$$

$$M_{21} = K_{21}(2\varphi_2 + \varphi_1 - 3\Psi_{21}) + \eta_{21}$$

$$M_{23} = K_{23}(2\varphi_2 + \varphi_3 - 3\Psi_{23}) + \eta_{23}$$

$$M_{12} = 0.667EI_1 \left(\varphi_2 - 3 \frac{\Delta_2}{3} \right)$$

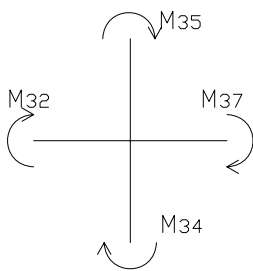
$$M_{21} = 0.667EI_1 \left(2\varphi_2 - 3 \frac{\Delta_2}{3} \right)$$

$$M_{23} = 1.185EI_1 (2\varphi_2 + \varphi_3)$$

$$\Sigma M_2 = 0$$

$$M_{23} + M_{21} = 0 \Rightarrow 3.704\varphi_2 + 1.185\varphi_3 - 0.667\Delta_2 = 0 \quad (1)$$

ČVOR 3.



$$\begin{aligned} M_{35} &= K_{35}(1.5 \varphi_3 - 1.5 \Psi_{35}) + \eta_{35} \\ M_{37} &= K_{37}(2 \varphi_3 + \varphi_7 - 3 \Psi_{37}) + \eta_{37} \\ M_{34} &= K_{34}(1.5 \varphi_3 - 1.5 \Psi_{34}) + \eta_{34} \\ M_{32} &= K_{32}(2 \varphi_3 + \varphi_2 - 3 \Psi_{32}) + \eta_{32} \end{aligned}$$

$$M_{35} = 0.75EI_1(1.5 \varphi_3 - 1.5(\frac{\Delta_1}{4} - \frac{\Delta_2}{4}))$$

$$M_{37} = 3.557(2 \varphi_3 + \varphi_7)$$

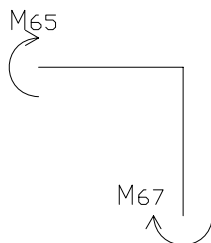
$$M_{34} = 0.75EI_1(1.5 \varphi_3 - 1.5 \frac{\Delta_2}{4})$$

$$M_{32} = 1.185EI_1(2 \varphi_3 + \varphi_2)$$

$$\Sigma M_3 = 0$$

$$M_{35} + M_{37} + M_{34} + M_{32} = 0 \Rightarrow 1.185 \varphi_2 + 11.72 \varphi_3 + 3.557 \varphi_7 - 0.281 \Delta_1 = 0 \quad (2)$$

ČVOR 6.



$$\begin{aligned} M_{65} &= K_{65}(1.5 \varphi_6 - 1.5 \Psi_{65}) + \eta_{65} \\ M_{67} &= K_{67}(2 \varphi_6 + \varphi_7 - 3 \Psi_{67}) + \eta_{67} \end{aligned}$$

$$\eta_{65} = \frac{ql^2}{8} = \frac{4 \cdot 6^2}{8} = 18$$

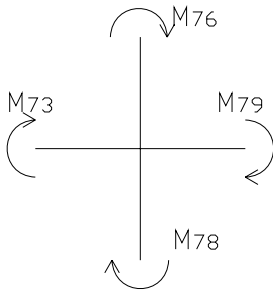
$$M_{65} = 5.335EI_1 \cdot 1.5 \varphi_6 + 18$$

$$M_{67} = 0.5EI_1(2 \varphi_6 + \varphi_7 - 3(\frac{\Delta_1}{4} - \frac{\Delta_2}{4}))$$

$$\Sigma M_6 = 0$$

$$M_{65} + M_{67} = 0 \Rightarrow 9.003 \varphi_6 + 0.5 \varphi_7 - 0.375 \Delta_1 + 0.375 \Delta_2 + \frac{18}{EI_1} = 0 \quad (3)$$

ČVOR 7.



$$\begin{aligned} M_{76} &= K_{76}(2\varphi_7 + \varphi_6 - 3\Psi_{76}) + \eta_{76} \\ M_{79} &= K_{79}(2\varphi_7 + \varphi_9 - 3\Psi_{79}) + \eta_{79} \\ M_{78} &= K_{78}(1.5\varphi_7 - 1.5\Psi_{78}) + \eta_{78} \\ M_{73} &= K_{73}(2\varphi_7 + \varphi_3 - 3\Psi_{73}) + \eta_{73} \end{aligned}$$

$$M_{76} = 0.5EI_1 (2\varphi_7 + \varphi_6 - 3(\frac{\Delta_1}{4} - \frac{\Delta_2}{4}))$$

$$\eta_{79} = -\frac{ql^2}{12} = -\frac{4 \cdot 5^2}{12} = -8.333$$

$$M_{79} = 1.852EI_1 (2\varphi_7 + \varphi_9) - 8.333$$

$$M_{78} = 0.75EI_1 (1.5\varphi_7 - 1.5\frac{\Delta_2}{4})$$

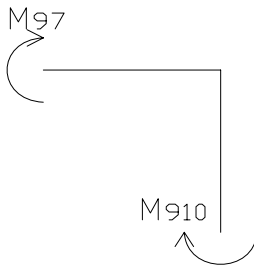
$$M_{73} = 3.557EI_1 (2\varphi_7 + \varphi_3)$$

$$\Sigma M_7 = 0$$

$$M_{76} + M_{79} + M_{75} + M_{73} = 0 \Rightarrow$$

$$3.557\varphi_3 + 0.5\varphi_6 + 12.929\varphi_7 + 1.852\varphi_9 - 0.375\Delta_1 + 0.094\Delta_2 - \frac{8.333}{EI_1} = 0 \quad (4)$$

ČVOR 9.



$$M_{109} = K_{109}(\varphi_6 - 3\Psi_{910})$$

$$M_{97} = K_{97}(2\varphi_9 + \varphi_7 - 3\Psi_{97}) + \eta_{97}$$

$$M_{910} = K_{910}(2\varphi_9 + \varphi_{10} - 3\Psi_{910}) + \eta_{910}$$

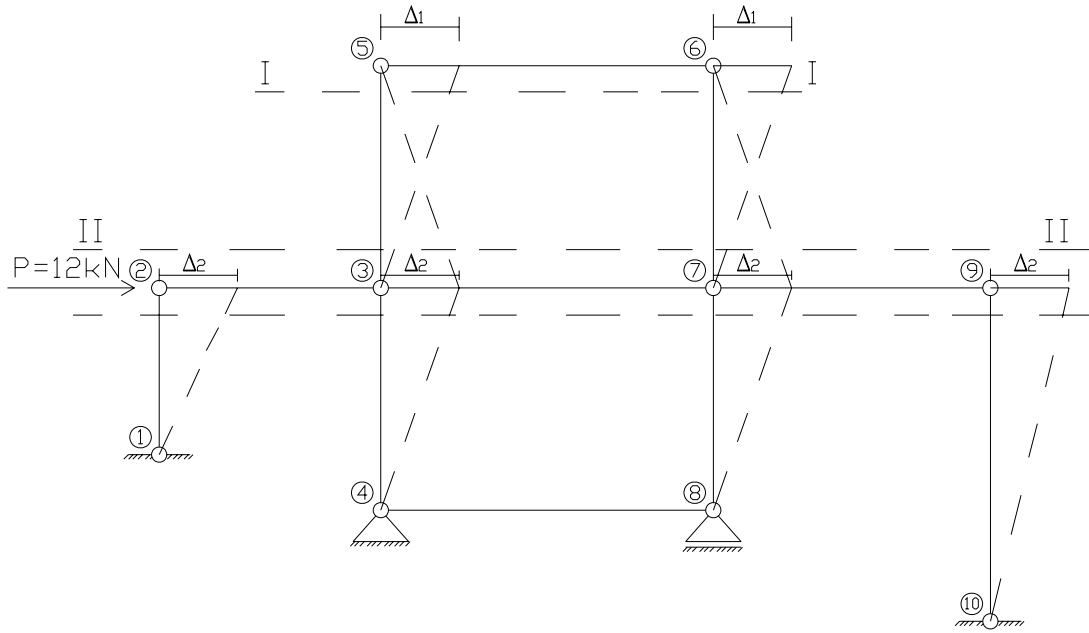
$$\eta_{97} = \frac{ql^2}{12} = \frac{4 \cdot 5^2}{12} = 8.333$$

$$M_{97} = 1.852EI_1 (2\varphi_9 + \varphi_7) + 8.333$$

$$M_{910} = 1.543EI_1 (2\varphi_9 - 3\frac{\Delta_2}{6})$$

$$\Sigma M_9 = 0$$

$$M_{97} + M_{910} = 0 \Rightarrow 1.852\varphi_7 + 6.79\varphi_9 - 0.772\Delta_2 + \frac{8.333}{EI_1} = 0 \quad (5)$$



Presjek I-I

$$\Sigma R_o - \frac{M_i + M_k}{l_{ik}} = T \quad R_o = 0, T = 0$$

$$\Sigma \frac{M_i + M_k}{l_{ik}} = \frac{M_{53} + M_{35}}{4} + \frac{M_{67} + M_{76}}{4}$$

$$\frac{(1.125\varphi_3 - 0.281\Delta_1 + 0.281\Delta_2)EI_1}{4} + \frac{(\varphi_6 + 0.5\varphi_7 - 0.375\Delta_1 + 0.375\Delta_2 + \varphi_7 + 0.5\varphi_6 - 0.375\Delta_1 + 0.375\Delta_2)EI_1}{4} = 0$$

$$-0.281\varphi_3 - 0.375\varphi_6 - 0.375\varphi_7 + 0.258\Delta_1 - 0.258\Delta_2 = 0 \quad (6)$$

Presjek II-II

$$R_o = 0, T = 12\text{kN}$$

$$-\left(\frac{M_{21} + M_{12}}{3} + \frac{M_{34}}{4} + \frac{M_{78}}{4} + \frac{M_{910} + M_{109}}{6} - \frac{M_{35}}{4} - \frac{M_{76} + M_{67}}{4}\right) = 0$$

$$-\left(\frac{(1.334\varphi_2 - 0.667\Delta_2 + 0.667\varphi_2 - 0.667\Delta_2)EI_1}{3} + \frac{(1.125\varphi_3 - 0.281\Delta_2)EI_1}{4} + \right.$$

$$+ \frac{(1.125\varphi_7 - 0.281\Delta_2)EI_1}{4} + \frac{(3.086\varphi_9 - 0.772\Delta_2 + 1.543\varphi_9 - 0.772\Delta_2)EI_1}{6} -$$

$$\frac{(1.125\varphi_3 - 0.281\Delta_1 + 0.281\Delta_2)EI_1}{4} -$$

$$\left.\frac{(\varphi_7 + 0.5\varphi_6 - 0.375\Delta_1 + 0.375\Delta_2 + \varphi_6 + 0.5\varphi_7 - 0.375\Delta_1 + 0.375\Delta_2)EI_1}{4}\right) = 12$$

$$-0.667\varphi_2 + 0.375\varphi_6 + 0.094\varphi_7 - 0.772\varphi_9 - 0.258\Delta_1 + 1.10\Delta_2 - 12 = 0 \quad (7)$$

čvor	φ_2	φ_3	φ_6	φ_7	φ_9	Δ_1	Δ_2	slobodni članovi
2	3.704	1.185	0	0	0	0	-0.667	0
3	1.185	11.72	0	3.557	0	-0.281	0	0
6	0	0	9.003	0.5	0	-0.375	0.375	-18/EI ₁
7	0	3.557	0.5	12.292	1.852	-0.375	0.094	8.333/EI ₁
9	0	0	0	1.852	6.79	0	-0.772	-8.333/EI ₁
I-I	0	-0.281	-0.375	-0.375	0	0.258	-0.258	0
II-II	-0.667	0	0.375	0.094	-0.772	-0.258	1.10	12/EI ₁

$$\varphi_2 = 3.242 / EI_1$$

$$\varphi_3 = -0.275 / EI_1$$

$$\varphi_6 = -2.136 / EI_1$$

$$\varphi_7 = 1.061 / EI_1$$

$$\varphi_9 = 0.475 / EI_1$$

$$\Delta_1 = 15.658 / EI_1$$

$$\Delta_2 = 17.518 / EI_1$$

ČVOR 2.

$$M_{12} = -9.52 \text{ kNm}$$

$$M_{21} = -7.36 \text{ kNm} \quad (7.36 \text{ kNm})$$

$$M_{23} = 7.36 \text{ kNm}$$

$$\Sigma M_2 = 0 \quad 7.36 - 7.36 = 0$$

ČVOR 3.

$$M_{35} = 0.22 \text{ kNm}$$

$$M_{37} = 1.81 \text{ kNm}$$

$$M_{34} = -5.22 \text{ kNm} \quad (5.22 \text{ kNm})$$

$$M_{32} = 3.19 \text{ kNm} \quad (-3.19 \text{ kNm})$$

$$\Sigma M_3 = 0 \quad 0.22 + 1.81 + 3.19 - 5.22 = 0$$

ČVOR 6.

$$M_{65} = 0.91 \text{ kNm} \quad (-0.91 \text{ kNm})$$

$$M_{67} = -0.91 \text{ kNm}$$

$$\Sigma M_6 = 0 \quad 0.91 - 0.91 = 0$$

ČVOR 7.

$$M_{76} = 0.70 \text{ kNm} \quad (-0.70 \text{ kNm})$$

$$M_{79} = -3.52 \text{ kNm}$$

$$M_{78} = -3.74 \text{ kNm}$$

$$M_{73} = 6.56 \text{ kNm} \quad (-6.56 \text{ kNm})$$

$$\Sigma M_7 = 0 \quad 0.75 + 6.56 - 3.74 - 3.52 = 0$$

ČVOR 9.

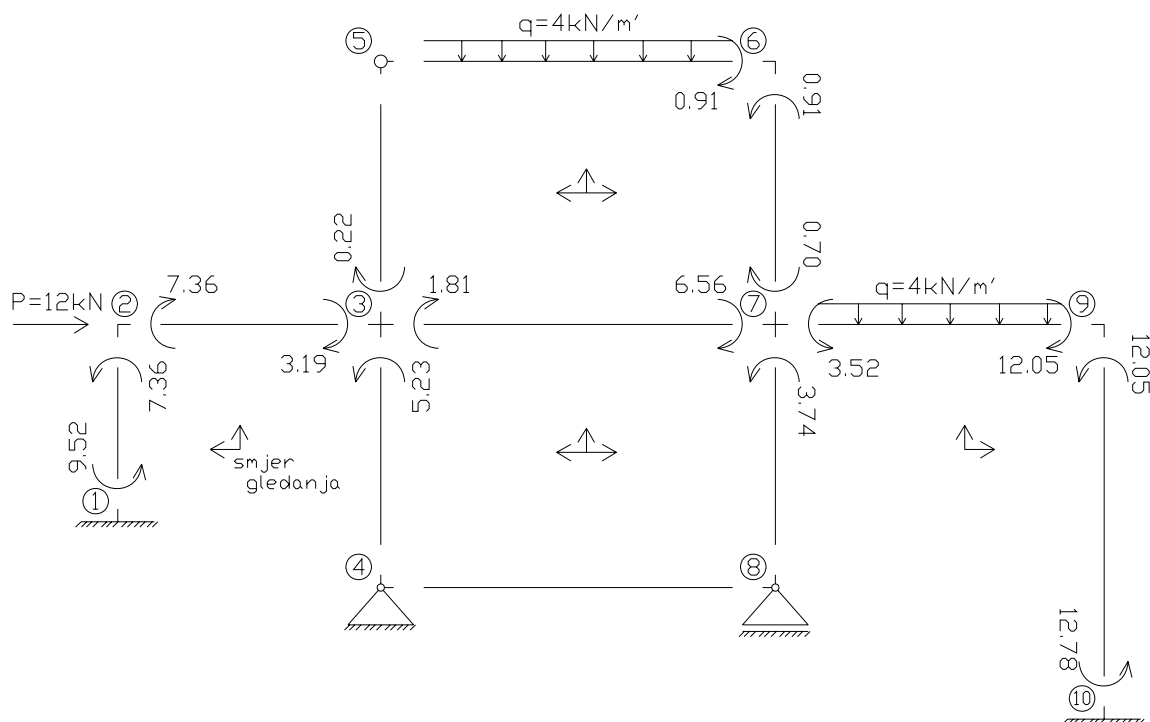
$$M_{910} = -12.05 \text{ kNm}$$

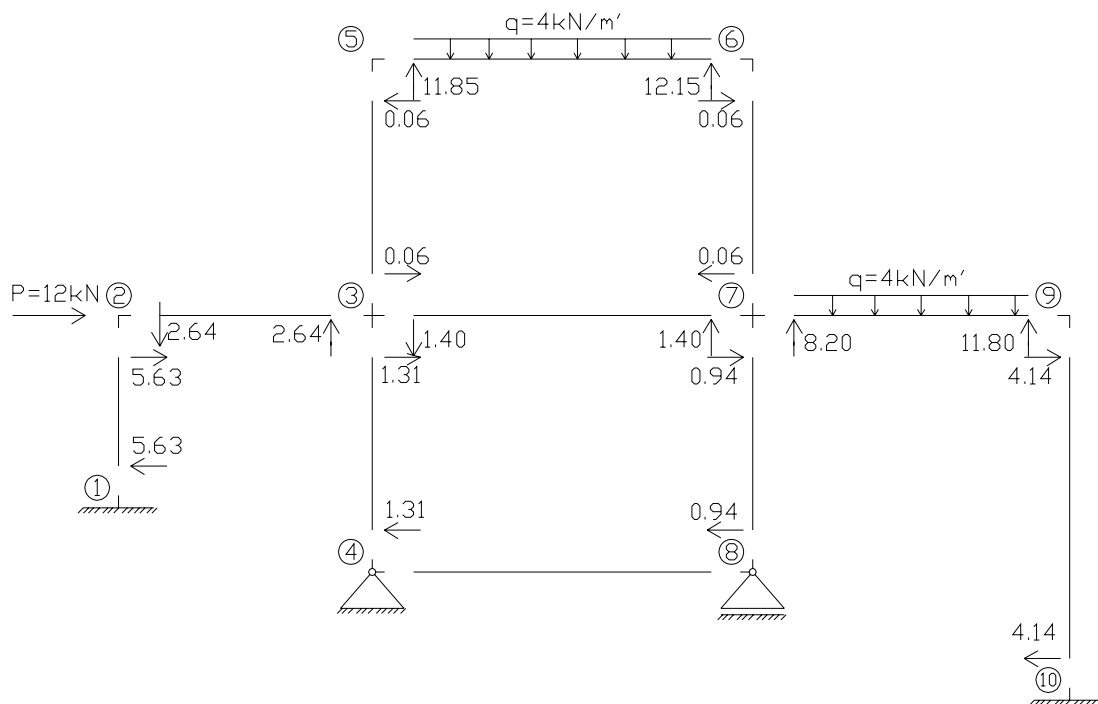
$$M_{97} = 12.05 \text{ kNm} \quad (-12.05 \text{ kNm})$$

$$M_{109} = -12.78 \text{ kNm} \quad (12.78 \text{ kNm})$$

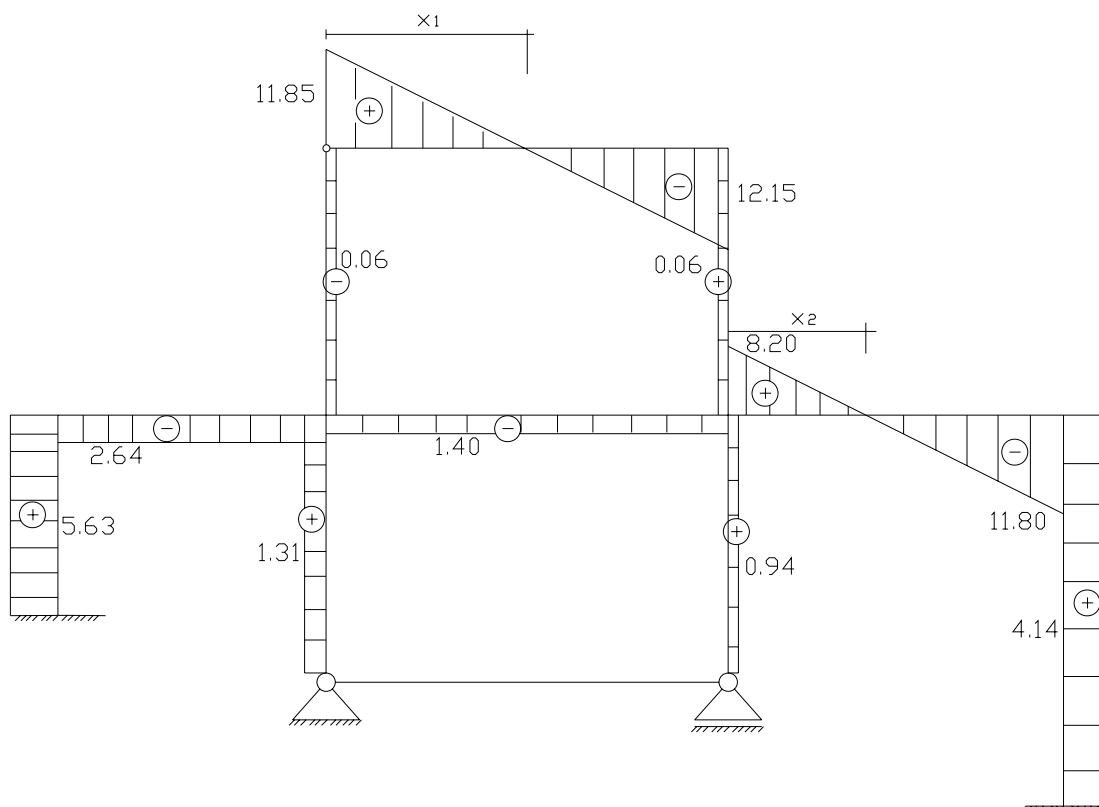
$$\Sigma M_9 = 0 \quad 12.05 - 12.05 = 0$$

šema opterecenja štapova





Dijagram transverzalnih sila



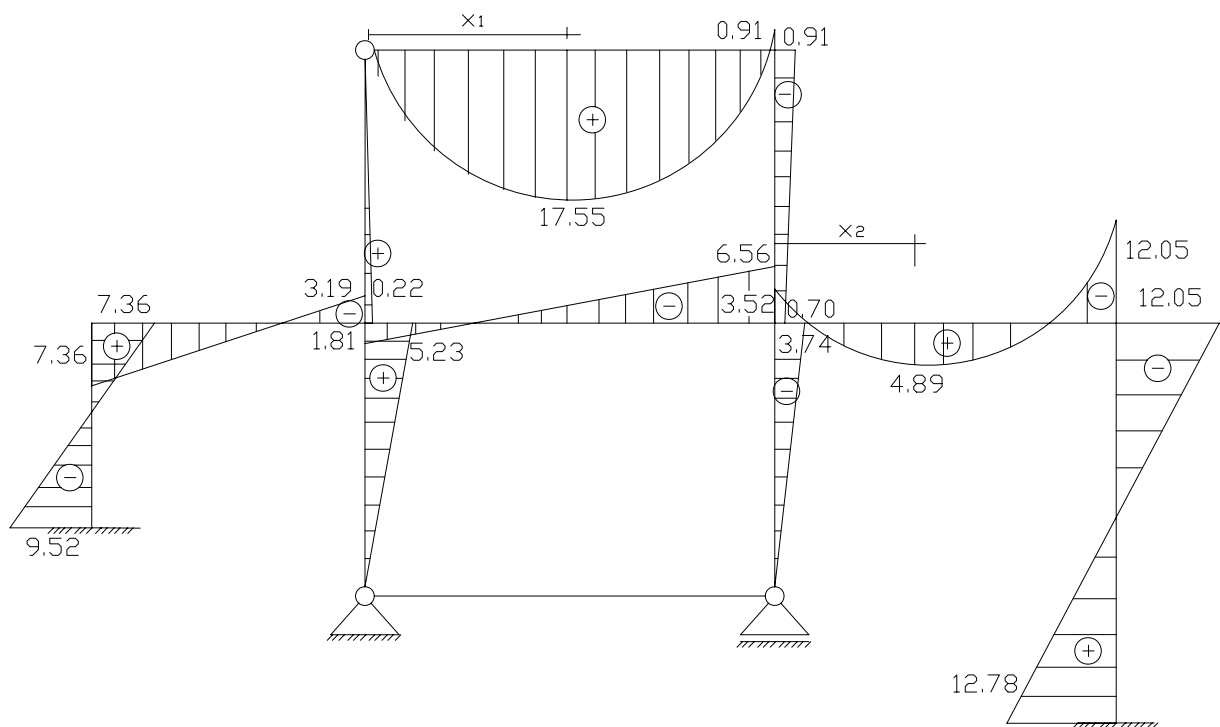
Dijagram momenata

$$\frac{11.85}{x_1} = \frac{12.15}{6 - x_1} \Rightarrow x_1 = 2.96\text{m}$$

$$M_{\max 1} = 11.85 \cdot 2.96 - \frac{4 \cdot 2.96^2}{2} = 17.55\text{kNm}$$

$$\frac{8.20}{x_2} = \frac{11.80}{5 - x_2} \Rightarrow x_2 = 2.05\text{m}$$

$$M_{\max 2} = -3.52 + 8.20 \cdot 2.05 - \frac{4 \cdot 2.05^2}{2} = 4.89\text{kNm}$$



Dijagram normalnih sila

