

$$EI \alpha = \frac{Pab}{6l}(b+l)$$

$$EI \beta = \frac{Pab}{6l}(a+l)$$

For Unit Force P

$$a=x$$

$$b=l-x$$

Then

$$\begin{aligned} EI \alpha &= \frac{x(l-x)}{6l}(l-x+l) = \frac{x}{6} \frac{l-x}{l} (2l-x) = \\ &= \frac{l}{6} \frac{x}{l} \frac{l-x}{l} \frac{2l-x}{l} l = \frac{l^2}{6} \frac{x}{l} \left(1 - \frac{x}{l}\right) \left(2 - \frac{x}{l}\right) \end{aligned}$$

$$\text{If } \xi = x/l$$

$$EI \alpha = \frac{l^2}{6} \xi (1 - \xi) (2 - \xi) = \frac{l^2}{6} \omega_D' \quad \text{where } \omega_D' = \xi (1 - \xi) (2 - \xi)$$

In the same way

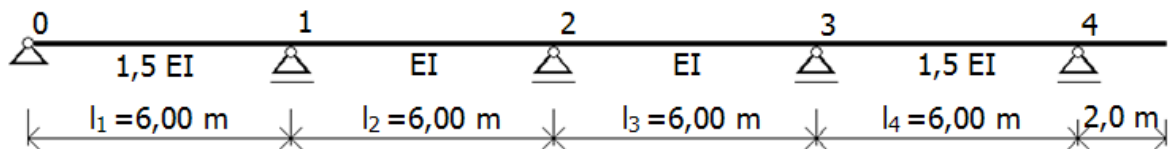
$$\begin{aligned} EI \beta &= \frac{x(l-x)}{6l}(x+l) = \frac{l}{6} \frac{x}{l} \frac{l-x}{l} \frac{x+l}{l} l = \\ &= \frac{l^2}{6} \frac{x}{l} \left(1 - \frac{x}{l}\right) \left(1 + \frac{x}{l}\right) = \frac{l^2}{6} \xi (1 - \xi) (1 + \xi) = \frac{l^2}{6} \omega_D \quad \text{where } \omega_D = \xi (1 - \xi) (1 + \xi) \end{aligned}$$

The values of coefficients ω_D and ω_D' are presented in the next table

x/l	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
ω_D	0.000	0.099	0.192	0.273	0.336	0.375	0.384	0.357	0.288	0.171	0.000
ω_D'	0.000	0.171	0.288	0.357	0.384	0.375	0.336	0.273	0.192	0.099	0.000

Example 1. For the next continuous beam with overhang determine the following:

- influence lines for support moments M_1 , M_2 and M_3 ,
- influence line for moment M_{26} (2nd span, section 6),
- influence line for shear force T_{26} ,
- influence line for support reaction R_2 .



Control rod stiffness: $EI_c = EI$

Reduced spans:

$$l'_1 = 6,0 \cdot \frac{1}{1,5} = 4,0 \text{ m}$$

$$l'_2 = 6,0 \cdot \frac{1}{1} = 6,0 \text{ m}$$

$$l'_3 = 6,0 \cdot \frac{1}{1} = 6,0 \text{ m}$$

$$l'_4 = 6,0 \cdot \frac{1}{1,5} = 4,0 \text{ m}$$

Clapeyron's equations:

$$2(l'_1 + l'_2) M_1 + l'_2 M_2 + 6 D_1' = 0$$

$$l'_2 M_1 + 2(l'_2 + l'_3) M_2 + l'_3 M_3 + 6 D_2' = 0$$

$$l'_3 M_2 + 2(l'_3 + l'_4) M_3 + 6 D_3' = 0$$

or:

$$20,0 M_1 + 6,0 M_2 = -6 D_1'$$

$$6,0 M_1 + 24,0 M_2 + 6,0 M_3 = -6 D_2'$$

$$6,0 M_2 + 20,0 M_3 = -6 D_3'$$

Unit Force $P=1.0$ in 1. span:

$$D_1' \neq 0$$

$$D_2' = 0 \text{ and } D_3' = 0$$

$$6D_1' = 6 \frac{l_1^2}{6} \omega_D \frac{1}{1.5} = \frac{l_1^2}{1.5} \omega_D$$

Unit Force $P=1.0$ in 2. span:

$$D_1' \neq 0$$

$$D_2' \neq 0$$

$$D_3' = 0$$

$$D_1' = \frac{l_2^2}{6} \omega_D \frac{1.0}{1.0} \quad \text{or} \quad 6D_1' = l_2^2 \omega_D$$

$$D_2' = \frac{l_2^2}{6} \omega_D \frac{1.0}{1.0} \quad \text{or} \quad 6D_2' = l_2^2 \omega_D$$

Unit Force $P=1.0$ in 3. span:

$$D_1' = 0$$

$$D_2' \neq 0$$

$$D_3' \neq 0$$

$$D_2' = \frac{l_3^2}{6} \omega_D \frac{1.0}{1.0} \quad \text{or} \quad 6D_2' = l_3^2 \omega_D$$

$$D_3' = \frac{l_3^2}{6} \omega_D \frac{1.0}{1.0} \quad \text{or} \quad 6D_3' = l_3^2 \omega_D$$

Unit Force $P=1.0$ in 4. span:

$$D_1' = 0$$

$$D_2' = 0$$

$$D_3' \neq 0$$

$$D_3' = \frac{l_4^2}{6} \omega_D \frac{1.0}{1.5} \quad \text{or} \quad 6D_3' = \frac{l_4^2}{1.5} \omega_D$$

Unit Force $P=1.0$ at the end of overhang:

$$D_1' = 0, D_2' = 0 \text{ and } D_3' = 0$$

The correction of 3. equation:

$$I_3' M_2 + 2(I_3' + I_4') M_3 + I_4' M_4 = 0$$

or

$$6,0M_2 + 20,0M_3 + 4,0(-2,0) = 0$$

$$6,0M_2 + 20,0M_3 = 8,0$$

Matrix of system:

20.0	6.0	0.0
6.0	24.0	6.0
0.0	6.0	20.0

Inverse Matrix of system:

0.054412	-0.014706	0.004412
-0.014706	0.049020	-0.014706
0.004412	-0.014706	0.054412

Influence lines for support moments M_1 , M_2 , M_3

Span 1

Span	Section	x	M_1	M_2	M_3
1	0	0.00	0.0000	0.0000	0.0000
1	1	0.60	-0.1293	0.0349	-0.0105
1	2	1.20	-0.2507	0.0678	-0.0203
1	3	1.80	-0.3565	0.0964	-0.0289
1	4	2.40	-0.4388	0.1186	-0.0356
1	5	3.00	-0.4897	0.1324	-0.0397
1	6	3.60	-0.5015	0.1355	-0.0407
1	7	4.20	-0.4662	0.1260	-0.0378
1	8	4.80	-0.3761	0.1016	-0.0305
1	9	5.40	-0.2233	0.0604	-0.0181
1	10	6.00	0.0000	0.0000	0.0000

Span 2

Span	Section	x	M_1	M_2	M_3
2	0	0.00	0.0000	0.0000	0.0000
2	1	0.60	-0.2825	-0.0842	0.0253
2	2	1.20	-0.4625	-0.1864	0.0559
2	3	1.80	-0.5548	-0.2928	0.0878
2	4	2.40	-0.5743	-0.3896	0.1169
2	5	3.00	-0.5360	-0.4632	0.1390
2	6	3.60	-0.4549	-0.4998	0.1499
2	7	4.20	-0.3458	-0.4855	0.1456
2	8	4.80	-0.2236	-0.4066	0.1220
2	9	5.40	-0.1034	-0.2494	0.0748
2	10	6.00	0.0000	0.0000	0.0000

Span 3

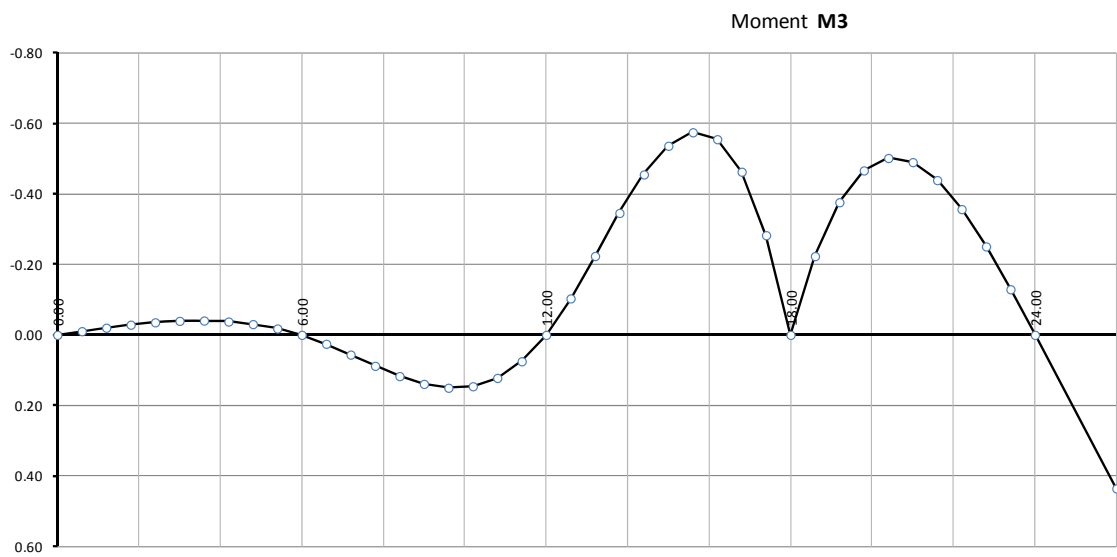
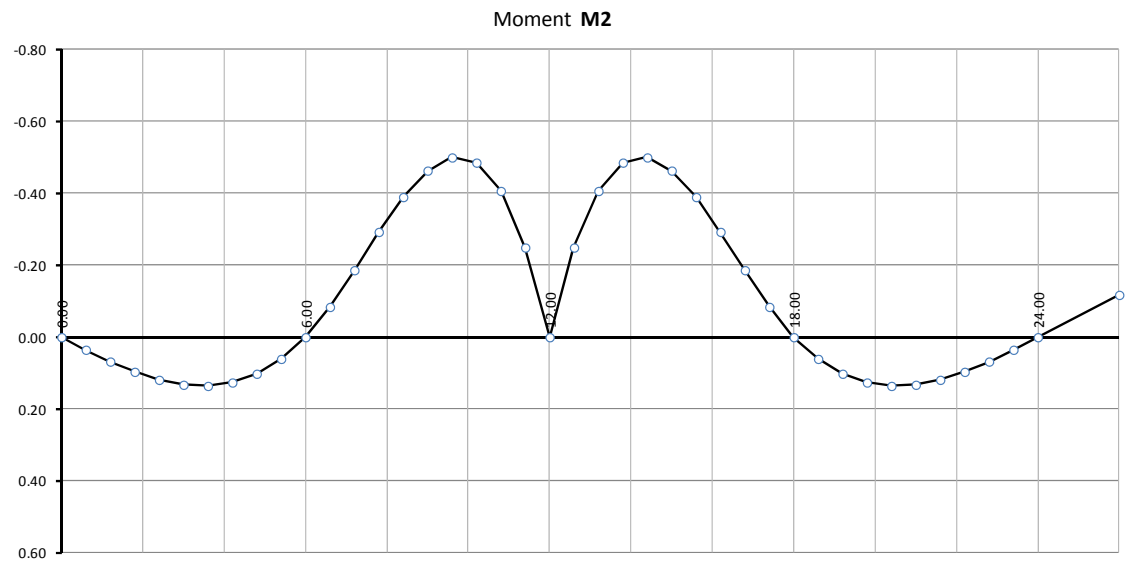
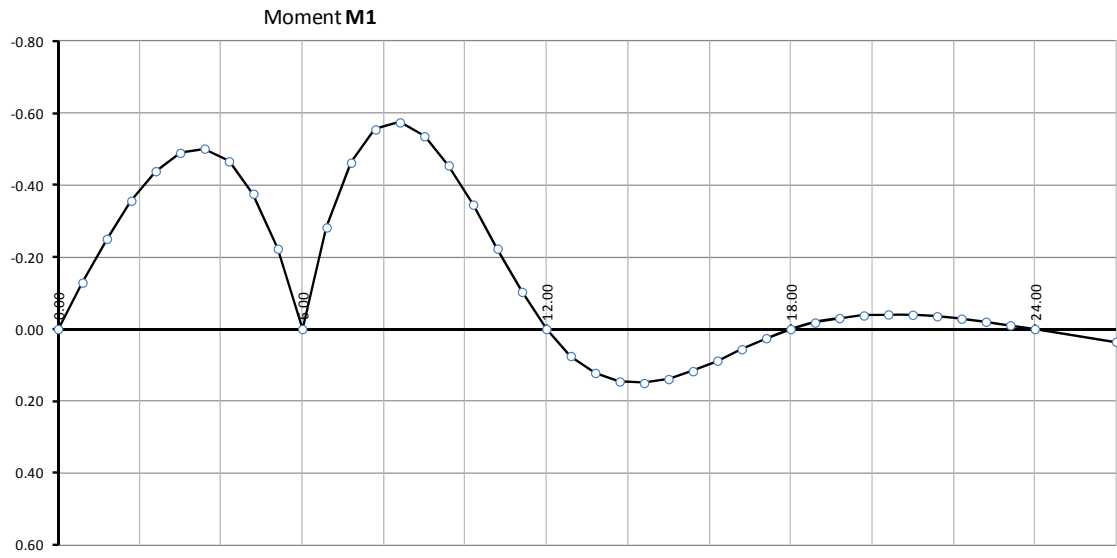
Span	Section	x	M ₁	M ₂	M ₃
3	0	0.00	0.0000	0.0000	0.0000
3	1	0.60	0.0748	-0.2494	-0.1034
3	2	1.20	0.1220	-0.4066	-0.2236
3	3	1.80	0.1456	-0.4855	-0.3458
3	4	2.40	0.1499	-0.4998	-0.4549
3	5	3.00	0.1390	-0.4632	-0.5360
3	6	3.60	0.1169	-0.3896	-0.5743
3	7	4.20	0.0878	-0.2928	-0.5548
3	8	4.80	0.0559	-0.1864	-0.4625
3	9	5.40	0.0253	-0.0842	-0.2825
3	10	6.00	0.0000	0.0000	0.0000

Span 4

Span	Section	x	M ₁	M ₂	M ₃
4	0	0.00	0.0000	0.0000	0.0000
4	1	0.60	-0.0181	0.0604	-0.2233
4	2	1.20	-0.0305	0.1016	-0.3761
4	3	1.80	-0.0378	0.1260	-0.4662
4	4	2.40	-0.0407	0.1355	-0.5015
4	5	3.00	-0.0397	0.1324	-0.4897
4	6	3.60	-0.0356	0.1186	-0.4388
4	7	4.20	-0.0289	0.0964	-0.3565
4	8	4.80	-0.0203	0.0678	-0.2507
4	9	5.40	-0.0105	0.0349	-0.1293
4	10	6.00	0.0000	0.0000	0.0000

Overhang

Span	Section	x	M ₁	M ₂	M ₃
5	0	0.00	0.0000	0.0000	0.0000
5	1	2.00	0.0353	-0.1176	0.4353



Influence line for moment M_{26} in section 6, span 2

$$M_{26} = M_{26}^0 + M_1 \cdot \left(1 - \frac{x_c}{l_2}\right) + M_2 \cdot \frac{x_c}{l_2}$$

$$M_{26} = M_{26}^0 + M_1 \cdot \left(1 - \frac{0,6 \cdot l_2}{l_2}\right) + M_2 \cdot \frac{0,6 \cdot l_2}{l_2}$$

$$M_{26} = M_{26}^0 + 0,4 \cdot M_1 + 0,6 \cdot M_2$$

Span 1

Span	Section	x	M_{26}^0	$0,4 \cdot M_1$	$0,6 \cdot M_2$	M_{26}
1	0	0.00		0.0000	0.0000	0.0000
1	1	0.60		-0.0517	0.0210	-0.0307
1	2	1.20		-0.1003	0.0407	-0.0596
1	3	1.80		-0.1426	0.0578	-0.0848
1	4	2.40		-0.1755	0.0712	-0.1044
1	5	3.00		-0.1959	0.0794	-0.1165
1	6	3.60		-0.2006	0.0813	-0.1193
1	7	4.20		-0.1865	0.0756	-0.1109
1	8	4.80		-0.1504	0.0610	-0.0894
1	9	5.40		-0.0893	0.0362	-0.0531
1	10	6.00		0.0000	0.0000	0.0000

Span 2

Span	Section	x	M_{26}^0	$0,4 \cdot M_1$	$0,6 \cdot M_2$	M_{26}
2	0	0.00	0.0000	0.0000	0.0000	0.0000
2	1	0.60	0.2400	-0.1130	-0.0505	0.0765
2	2	1.20	0.4800	-0.1850	-0.1118	0.1832
2	3	1.80	0.7200	-0.2219	-0.1757	0.3224
2	4	2.40	0.9600	-0.2297	-0.2338	0.4965
2	5	3.00	1.2000	-0.2144	-0.2779	0.7076
2	6	3.60	1.4400	-0.1819	-0.2999	0.9582
2	7	4.20	1.0800	-0.1383	-0.2913	0.6504
2	8	4.80	0.7200	-0.0894	-0.2440	0.3866
2	9	5.40	0.3600	-0.0414	-0.1496	0.1690
2	10	6.00	0.0000	0.0000	0.0000	0.0000

Span 3

Span	Section	x	M_{26}^0	$0,4 \cdot M_1$	$0,6 \cdot M_2$	M_{26}
3	0	0.00		0.0000	0.0000	0.0000
3	1	0.60		0.0299	-0.1496	-0.1197
3	2	1.20		0.0488	-0.2440	-0.1952
3	3	1.80		0.0583	-0.2913	-0.2330
3	4	2.40		0.0600	-0.2999	-0.2399
3	5	3.00		0.0556	-0.2779	-0.2224
3	6	3.60		0.0468	-0.2338	-0.1870
3	7	4.20		0.0351	-0.1757	-0.1405
3	8	4.80		0.0224	-0.1118	-0.0894
3	9	5.40		0.0101	-0.0505	-0.0404
3	10	6.00		0.0000	0.0000	0.0000

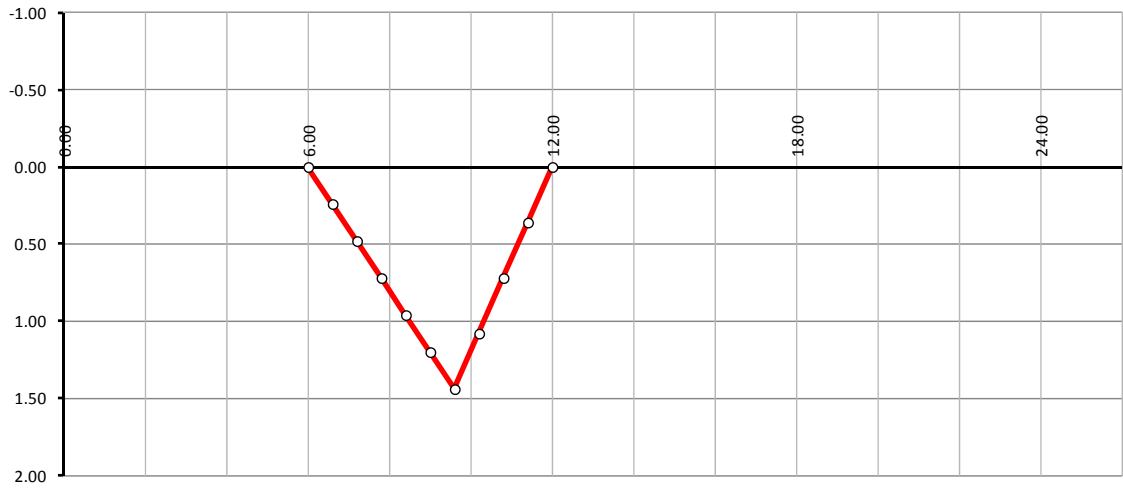
Span 4

Span	Section	x	M_{26}^0	$0,4 \cdot M_1$	$0,6 \cdot M_2$	M_{26}
4	0	0.00		0.0000	0.0000	0.0000
4	1	0.60		-0.0072	0.0362	0.0290
4	2	1.20		-0.0122	0.0610	0.0488
4	3	1.80		-0.0151	0.0756	0.0605
4	4	2.40		-0.0163	0.0813	0.0651
4	5	3.00		-0.0159	0.0794	0.0635
4	6	3.60		-0.0142	0.0712	0.0569
4	7	4.20		-0.0116	0.0578	0.0462
4	8	4.80		-0.0081	0.0407	0.0325
4	9	5.40		-0.0042	0.0210	0.0168
4	10	6.00		0.0000	0.0000	0.0000

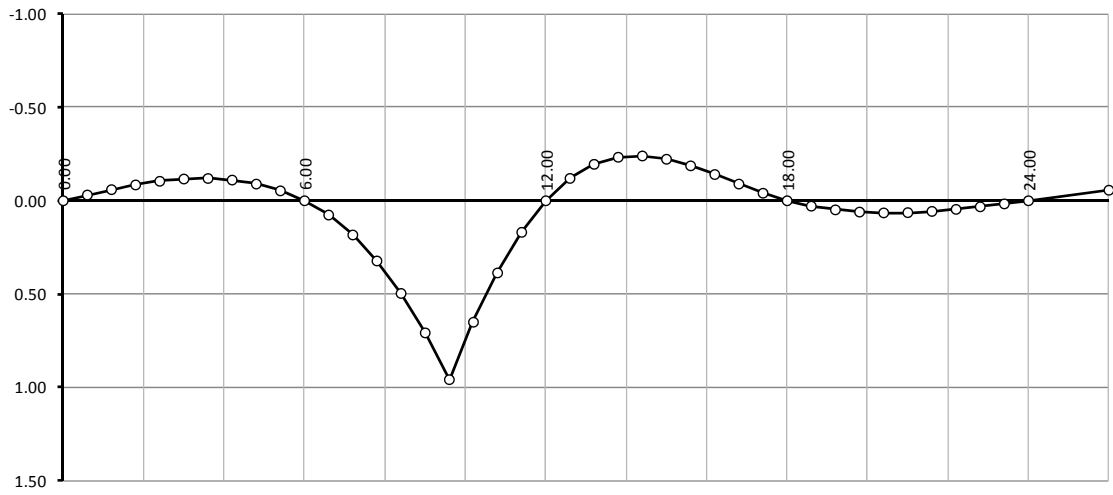
Overhang

Span	Section	x	M_{26}^0	$0,4 \cdot M_1$	$0,6 \cdot M_2$	M_{26}
5	0	0.00		0.0000	0.0000	0.0000
5	1	2.00		0.0141	-0.0706	-0.0565

Moment M_{26}^0



Moment M_{26}



Influence line for shear force T_{26} in section 6, span 2

$$T_{26} = T_{26}^0 + \frac{M_2 - M_1}{l_2}$$

$$T_{26} = T_{26}^0 + \frac{M_2 - M_1}{6}$$

Span 1

Span	Section	x	T_{26}^0	$(M_2 - M_1)/6$	T_{26}
1	0	0.00		0.0000	0.0000
1	1	0.60		0.0274	0.0274
1	2	1.20		0.0531	0.0531
1	3	1.80		0.0755	0.0755
1	4	2.40		0.0929	0.0929
1	5	3.00		0.1037	0.1037
1	6	3.60		0.1062	0.1062
1	7	4.20		0.0987	0.0987
1	8	4.80		0.0796	0.0796
1	9	5.40		0.0473	0.0473
1	10	6.00		0.0000	0.0000

Span 2

Span	Section	x	T_{26}^0	$(M_2 - M_1)/6$	T_{26}
2	0	0.00	0.0000	0.0000	0.0000
2	1	0.60	-0.1000	0.0331	-0.0669
2	2	1.20	-0.2000	0.0460	-0.1540
2	3	1.80	-0.3000	0.0437	-0.2563
2	4	2.40	-0.4000	0.0308	-0.3692
2	5	3.00	-0.5000	0.0121	-0.4879
2	6	3.60	-0.6000 0.4000	-0.0075	-0.6075 0.3925
2	7	4.20	0.3000	-0.0233	0.2767
2	8	4.80	0.2000	-0.0305	0.1695
2	9	5.40	0.1000	-0.0243	0.0757
2	10	6.00	0.0000	0.0000	0.0000

Span 3

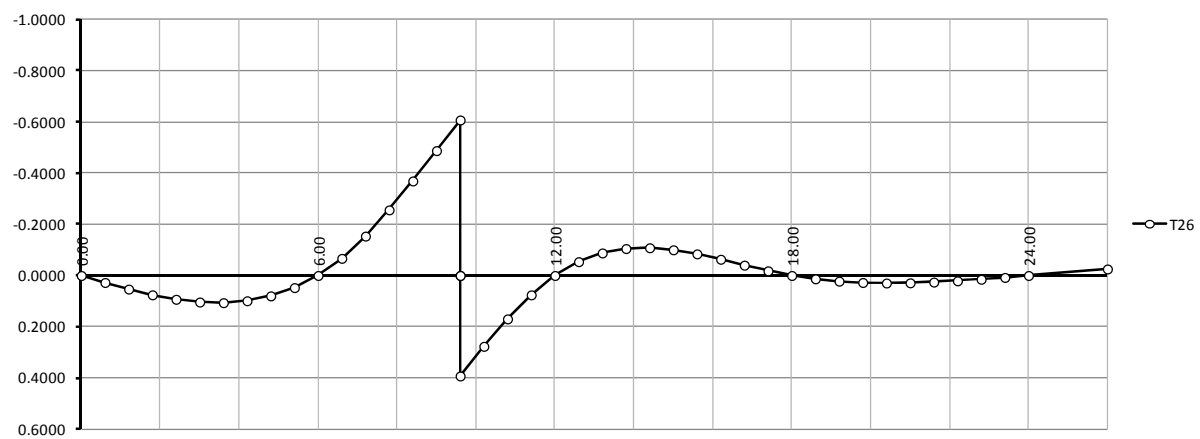
Span	Section	x	T_{26}^0	$(M_2 - M_1)/6$	T_{26}
3	0	0.00		0.0000	0.0000
3	1	0.60		-0.0540	-0.0540
3	2	1.20		-0.0881	-0.0881
3	3	1.80		-0.1052	-0.1052
3	4	2.40		-0.1083	-0.1083
3	5	3.00		-0.1004	-0.1004
3	6	3.60		-0.0844	-0.0844
3	7	4.20		-0.0634	-0.0634
3	8	4.80		-0.0404	-0.0404
3	9	5.40		-0.0182	-0.0182
3	10	6.00		0.0000	0.0000

Span 4

Span	Section	x	T_{26}^0	$(M_2 - M_1)/6$	T_{26}
4	0	0.00		0.0000	0.0000
4	1	0.60		0.0131	0.0131
4	2	1.20		0.0220	0.0220
4	3	1.80		0.0273	0.0273
4	4	2.40		0.0294	0.0294
4	5	3.00		0.0287	0.0287
4	6	3.60		0.0257	0.0257
4	7	4.20		0.0209	0.0209
4	8	4.80		0.0147	0.0147
4	9	5.40		0.0076	0.0076
4	10	6.00		0.0000	0.0000

Overhang

Span	Section	x	T_{26}^0	$(M_2 - M_1)/6$	T_{26}
5	0	0.00		0.0000	0.0000
5	1	2.00		-0.0255	-0.0255



Influence line for reaction in support 2

$$R_2 = {}_d T_2^0 - {}_l T_2^0 + \frac{M_1}{l_2} - \left(\frac{1}{l_2} + \frac{1}{l_3}\right) \cdot M_2 + \frac{M_3}{l_3} = R_2^0 + \frac{M_1}{l_2} - \left(\frac{1}{l_2} + \frac{1}{l_3}\right) \cdot M_2 + \frac{M_3}{l_3}$$

$$R_2 = R_2^0 + \frac{M_1}{6} - \frac{M_2}{3} + \frac{M_3}{6} = R_2^0 + \frac{M_1 - 2 \cdot M_2 + M_3}{6} = R_2^0 + R_2^M$$

Span 1

Span	Section	x	R_2^0	R_2^M	R_2
1	0	0.00		0.0000	0.0000
1	1	0.60		-0.0349	-0.0349
1	2	1.20		-0.0678	-0.0678
1	3	1.80		-0.0964	-0.0964
1	4	2.40		-0.1186	-0.1186
1	5	3.00		-0.1324	-0.1324
1	6	3.60		-0.1355	-0.1355
1	7	4.20		-0.1260	-0.1260
1	8	4.80		-0.1016	-0.1016
1	9	5.40		-0.0604	-0.0604
1	10	6.00		0.0000	0.0000

Span 2

Span	Section	x	R_2^0	R_2^M	R_2
2	0	0.00	0.0000	0.0000	0.0000
2	1	0.60	0.1000	-0.0148	0.0852
2	2	1.20	0.2000	-0.0056	0.1944
2	3	1.80	0.3000	0.0198	0.3198
2	4	2.40	0.4000	0.0536	0.4536
2	5	3.00	0.5000	0.0882	0.5882
2	6	3.60	0.6000	0.1158	0.7158
2	7	4.20	0.7000	0.1285	0.8285
2	8	4.80	0.8000	0.1186	0.9186
2	9	5.40	0.9000	0.0784	0.9784
2	10	6.00	1.0000	0.0000	1.0000

Span 3

Span	Section	x	R_2^0	R_2^M	R_2
3	0	0.00	1.0000	0.0000	1.0000
3	1	0.60	0.9000	0.0784	0.9784
3	2	1.20	0.8000	0.1186	0.9186
3	3	1.80	0.7000	0.1285	0.8285
3	4	2.40	0.6000	0.1158	0.7158
3	5	3.00	0.5000	0.0882	0.5882
3	6	3.60	0.4000	0.0536	0.4536
3	7	4.20	0.3000	0.0198	0.3198
3	8	4.80	0.2000	-0.0056	0.1944
3	9	5.40	0.1000	-0.0148	0.0852
3	10	6.00	0.0000	0.0000	0.0000

Span 4

Span	Section	x	R_2^0	R_2^M	R_2
3	0	0.00		0.0000	0.0000
3	1	0.60		-0.0604	-0.0604
3	2	1.20		-0.1016	-0.1016
3	3	1.80		-0.1260	-0.1260
3	4	2.40		-0.1355	-0.1355
3	5	3.00		-0.1324	-0.1324
3	6	3.60		-0.1186	-0.1186
3	7	4.20		-0.0964	-0.0964
3	8	4.80		-0.0678	-0.0678
3	9	5.40		-0.0349	-0.0349
3	10	6.00		0.0000	0.0000

Overhang

Span	Section	x	R_2^0	R_2^M	R_2
5	0	0.00		0.0000	0.0000
5	1	2.00		0.1176	0.1176

